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The Impulse Responses of Capital, Consumption, and Prices in the Stochastic Ramsey Model with an Infinite Time Horizon Employing Value Function Iteration and Linear Interpolation

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***Abstract**: In this paper, we use value function iteration and linear interpolation to solve an example of a stochastic, infinite-horizon Ramsey model, which is one kind of dynamic general equilibrium (DGE) model. To obtain an approximation of the policy function's solution, we pick a grid of capital investments for each period, set the starting value of the value function, build an iterative loop to compute the new value using interpolation, and establish the selection criteria and stop conditions. We model transient shocks to total factor productivity (TFP) and capture the impulsive reactions of consumption, capital, and factor pricing, with the algorithm anticipated to converge in "x" iterations. This concept is helpful and inspiring for assessing financial risk avoidance and dealing with and better comprehending potential shocks.*

***Keywords:**Value Function Iteration, Linear Interpolation, Stochastic Infinite Horizon, Ramsey Model*

***JEL:** C35, C51, C61*

1. Introduction

The interplay of capital, consumption, and factor prices, as well as the evaluation of their volatility, is a recurrent issue in financial markets that influences several activities, including financial derivatives, international commerce, exchange rate fluctuations, and cross-border investments. In 2022, geostrategic unpredictability and the persistence of the pandemic will increase the negative effect of external shocks on the economic externalities. And financial risk avoidance requires an understanding of how to anticipate potential shocks and evaluate the horizon over which they stabilize. The purpose of this paper is to employ discrete state space techniques to solve a dynamic general equilibrium (DGE) model in economics. Since the Ramsey model is a workhorse example of a DGE model, we use it to illustrate our approach. We choose a grid of capital investments for each period, establish the starting value of the value function, create an iterative loop to compute the new value using interpolation, and define the selection criteria and stop conditions. Finally, we model transient shocks to total factor productivity (TFP) and identify the impulsive responses of consumption, capital, and factor pricing.

The remainder of this paper is structured as follows: we begin by reviewing the model and its implementation by prior researchers. We build up the model according to Heer and Maussner (2011) and discuss the value function iteration and linear interpolation. Then, we construct the model calibration and the policy function that results. We assess the model using impulsive reactions to a transient shock in total factor production, and we finish with a discussion of the model's limits and development opportunities. We discover that discrete state space approaches can represent the value function and policy function of our stochastic Ramsey model properly. In terms of capital, consumption, and factor prices, we discover that our model can replicate empirical data in broad strokes.

2. Literature Review

The traditional Ramsey growth model is an economic growth model that has grown through numerous generations of modifications (Guerrini, 2010), with an exogenous labor-augmenting technology advancement based on Ramsey (1928)'s work and subsequently expanded by Cass (1965). Johnson *et al.* (1993) discuss the trade-off between accuracy and time needed by various solutions, while Heer and Maussani (2011) provide a complete summary of the numerical approaches used to solve the Ramsey Model. The enhanced Bayesian method is also used in value iteration algorithms (Engel *et al.*, 2003). For this research, we depend on one of the solutions offered by Heer and Maussner (2011), considering it an adequate resource for our theoretical explanation and optimization technique.

Modern scholars have studied at the junction of computational economics and macroeconomic theory to solve this stochastic model. The value function iteration is often used to evaluate the precision of numerical algorithms (Taylor & Uhlig, 1990). The optimum fiscal policy technique locates the function around zero such that quadratic convergence may be attained (Puterman & Brumelle, 1979); Zhu (1992) describes this method in detail. The dynamic planning approach is based on a very fine capital grid, which seems to be superior to other algorithms in terms of speed, accuracy, and ease of installation (Christiano & Fisher, 2000). Jiang and Powell (2015) provide the optimum value function for three application domains: optimal stopping, energy storage/allocation, and diabetic glycemic management.

Roberto Chang(1998)'scredible monetary model gives a computer method that converges to a set of time-consistent outcomes, which has influenced interest rate policy (Campbell & Weber, 2021), food price fluctuations (Cato & Chang, 2015), and sovereign debt research (Engel & Park, 2022).

Additionally, stochastic dynamic selection procedures and value function iteration may simulate the risk of inter-temporal family financial choices (Elbers *et al.*, 2009). Erosa and Ventura (2002) construct a threedimensional variables household interest optimization. This model was applied to environmental economics by Kelly and Kolsta (2001), who extend the classical model to incorporate the environment and pollutant produced. Traeger (2014) change the Bellman model to the DICE model in order to solve for an indefinite time horizon at arbitrary time steps and to quantify the economic systemic consequences of climate change, energy conservation, and emission reduction. Mirman *et al.* (2008) create a new class of monotonic iterative techniques that provide a qualitative theory for models of public policy, fiat currency, monopolistic competition, externalizes, and also other nonconvexities of output intensity.

3. The Model

In 1928, Frank Ramsey presented the topic of how much of a nation's revenue it should be saving (Heer and Maussner, 2011). Since then, a number of economists have been motivated to focus their research on the macro economy by means of dynamic optimization issues (Becker *et al.*, 1989). To start, we instantiate our Cobb-Douglas type creation function y_t at time t, which is as follows:

$$\begin{cases} y_t = A_t f(K_t) \\ f(K_t) = K_t^{\alpha} \end{cases} \tag{1}$$

In this scenario, "A" refers to any external factor that boosts overall production, such as the advancement of technology. This idea of technical advancement, which is rather general and nebulous, does not apply to one particular technology in particular. Calculations may be made to determine the pace of technical advancement during a certain long-term historical period by comparing it to the natural growth rate of the average production throughout that time period. The invested capital K_t is employed to produce output; as a result, the capital is used up and depreciated, bringing the total amount of $(1-\delta)\ K_t$ in the next period. The output is either used for consumption C_t ,or put back into production as an investment K_{t+1}. This is our budget constraint.

$$y_t + (1-\delta)K_t \geq C_t + K_{t+1} \tag{2}$$

The utility $u(C_t)$ of the agent at time t is determined by consumption using an isoelastic (CES) function that is concave and growing in C(Benhabib and Rustichini, 1994):

$$u(C_t) = \frac{C_t^{1-\eta} - 1}{1-\eta}, \eta > 0 \tag{3}$$

But this involves choosing C at each instant of time ratherthan choosing a finite set of variables, as in the standard maximization problems. The household actually chooses C at eachpoint in time. In other words, it chooses infinite C_t. By picking the consumption route of, the family works toward the goal of maximizing the lifetime utility within the confines of the economy's resource restriction. On the other hand, this requires picking C at every single moment in time as opposed to picking a limited number of variables, as is done in the conventional maximizing issues. In practice, the family always goes with option C at any given pointin time. In other words, it opts for an endless amount of C_t.

$$\max U_t = \sum_{t=1}^{\infty} \beta^{t-1} u(C_t), \beta \in (0,1) \tag{4}$$

which is contingent upon the budget limitation outlined in the previous Equation (2) and necessitates that at any given time t,

$$C_t \geq 0, K_{t+1} \geq 0 \tag{5}$$

3.1. Dynamics and Equilibrium

The Euler equation encapsulates the characteristics of the solution to the deterministic Ramsey problem:

$$\frac{u'(f(K_t) - K_{t+1})}{u'(f(K_{t+1}) - K_{t+2})} - \beta f'(K_{t+1}) = 0 \tag{6}$$

In order to provide a concise explanation of the dynamics shown by the Ramsey model, we make use of the approach represented by the phase diagram in Figure 1. In order to maintain the readability of this report, we will forgo going into detail about the Euler equation and will instead concentrate on the ramifications of this equation as shown by our phase diagram.

Because there is a finite amount of resources, the capital stock K in the dynamic Ramsey model has a limit, which is an important aspect of the

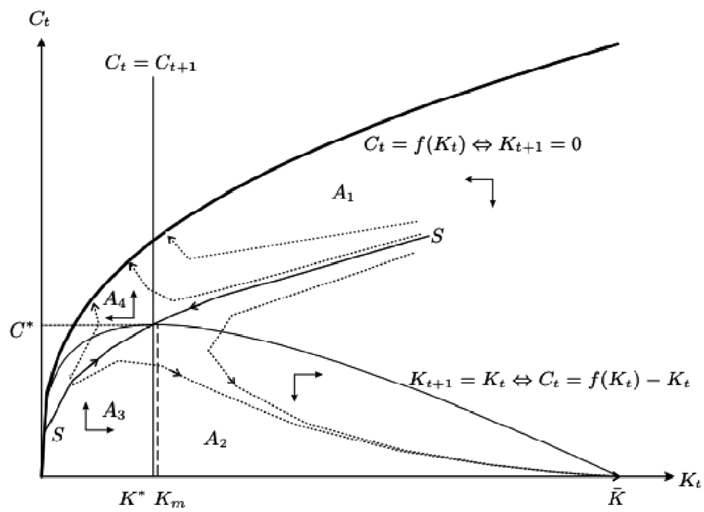


Figure 1: Phase Diagram of the Infinite Horizon Ramsey Model

model. If K is more than a certain value $\bar{K}$, then the output that it generates will be less than the production that is required to keep K at its present level, even if consumption is reduced to zero; this will result in a decrease in capital level in the subsequent instance.

The graphic provides a representation of the highest limit of our capital stock $\bar{K}$, which is visible to us. The point is located at the end of the curve that represents the amount of consumption necessary to keep a certain level of capital at its current level. It represents C_t such that $K_{t+1} = K_t$. At that point, it reaches its maximal consumption K_m. Despite the evident advantages, the system does not achieve its equilibrium at K_m but rather a bit to the left of K_m, at K^*. At K^*, the level of consumption does not go up since the marginal product of capital $f'K^* = 1/\beta$ is constant. As a result, the system arrives at K^*, C^* when it has reached its stationary equilibrium .

We are also able to demonstrate that the only possible time route of consumption is on the curve that is represented by the saddle path. This is the case for any given quantity of beginning capital. For values of $K < K^*$, consumption will adjust itself to a level that allows capital to grow until it reaches K^*. If $K > K^*$, the system will again converge to the stationary equilibrium following the saddle path. Let us suppose that there is a policy function $h(K_t)$ that can be thought of as a decision rule and that has the following relationship between the capital stock at each instant and the optimum consumption. During this experiment, we will do our best to estimate this policy function using numbers.

$$C_t = Af(K_t) - h(K_t) \tag{7}$$

3.2. Stochastic Total Factor Productivity

The random variable in this DGE originates from the total productivity factor, and we define A as a totally exogenous shock that is controlled by a stationary stochastic process. The total productivity factor is where the random variable comes from (Carlson *et al.*, 2012). In particular, log$A$ changes according to a process known as AR(1), which is based on the benchmark model described in Heer and Maussner (2011):

Consequently, the total factor productivity is unknown, and its divergence from unity affects total output. If A is greater than 1, production would increase, leading to a rise in total consumption and capital expenditures in the subsequent period.Similarly, if $A<1$, production would decrease, leading to decreased consumption and investment in the next period.The stationary equilibrium K^* and C^* would be different for different levels of A. In fact, given initial values A_0 and K_0, nowwe have an expected value of utility from future periods which needs to be maximized by choosing the optimal level ofinitial consumption C_0, or next-period investment K_{t+1}.

$$\max E_0[\sum_{t=0}^{\infty} \beta^t u(C_t)] \tag{8}$$

Therefore, we have

$$\begin{aligned} &K_{t+1} + C_t \le A_t f(K_t) + (1-\delta)K_t \\ &0 \le C_t \\ &0 \le K_{t+1} \end{aligned} \tag{9}$$

We use the method ofTauchen (1986) to derive a matrix of transition probabilities of A from one value to anotherdenoted byso that the solution concepts identified in the next section can be applied effectively.

4. Value Function Iteration and Linear Interpolation

To find the policy function we require a recursive formulation of the Ramsey problem. To do so we must assume that, given A and an initial capital stock K_0, we already know the optimal time path of capital (and therefore consumption). This series of optimal capital investments is denoted by $(K_0^*)_{t=1}^{\infty} = (K_1^*, K_2^*, ...)$. Over an infinite lifetime, the utility fromconsumption by following this optimal path is given by a value function $v(K_0)$:

$$v(K_0) = \max u(f(K_t) - K_1^*) + \sum_{t=1}^{\infty} \beta^t u(f(K^*) - K_{t+1}^*)$$

Changing notation slightly, in the stochastic case at any instant for a given level of TFP A and capital K, we can find thenext period investmentusing the value function:

$$v(K, A) = \max u(A, K, K') + \beta E[v(K', Z') \mid Z]$$

where u is the current period utility of consumption $C = Zf(K) + (1-\delta)K - K'$. We can further adapt the equation above to our model specifications to obtain the Bellman equation:

$$V(K, A) = \max \frac{(AK^{\alpha} + (1-\delta)K - K')^{1-\eta} - 1}{1-\eta} + \beta \sum_{l=1}^{m} p_{jl} v(K') \quad (10)$$

We begin by choosing an error tolerance η. Next we discretize the state space by constructing a grid for capital: $K = K_1, K_2, K_3, \ldots, K_n$ and the initial guess for the value function's choice of K' is that $K' = 0$. We then iterate over the valuefunction, penalizing negative values of K. We recognize convergence when the K' values stop changing for subsequent iterations. Since the technique that was used to derive the value function V alliteratively from the Bellman equation can be found clearly defined in Heer and Maussner (2011), it is not particularly productive to try to duplicate the algorithm that was utilized. In light of the fact that the discrete values that are supplied in the grid are insufficient, we interpolated a policy function at each step by making use of the grid points that were accessible, and then we updated the subsequent iterations appropriately. As a result, the values of K' that we converged upon can be different from the ones that were picked for the original grid.

4.1. Model Calibration

In light of the fact that model calibration is essential to the precision and effectiveness of our algorithm, we consulted the relevant research in order to choose estimates of model parameters that have been previously validated. Although many macroeconomic experts feel that the capital share is somewhere between 0.3 and 0.4 (Auerbach, 1979), the question is still contentious due to the possibility that this ratio may vary in the long term. When it comes to the subjective discounted factor known as β, the value is often either 0.9 or 0.95 in the majority of macroeconomic models. The annual depreciation of capital is estimated at 10 percent by the majority of economists (Nadiri and Prucha, 1996). It is not immediately clear how to estimate the value of the CRRA parameter (Dacy and Hasanov, 2011). Following a process of trial and error, we have determined that the parameter values that Heer and Maussner (2011)used are trustworthy, and therefore we set to α =0.27, β = 0.994, δ =0.011, η = 2, ρ = 0.90 and σ = 0.0072.

Since the exogenous shock A follows a stationary stochastic process, expectations and probabilities should be takeninto account. We denote A as a $q \times 1$ vector representing possible realizations. And we also denote $P = (p_{ji})$ as a $q \times q$ matrix, whose row j and column i element is the probability of transitioning to state j from state i. We use themethod by Tauchen (1986) to derive the values of A as well as transition probabilities. We compute a probability matrix of:

$$P = \begin{pmatrix} 0.96677 & 0.03322 & 6.6e^{-11} \\ 0.01089 & 0.97821 & 0.01089 \\ 6.6e^{-11} & 0.03322 & 0.96677 \end{pmatrix} \tag{11}$$

The whole expected probability of total factor productivity (TFP) next period always equal to 1, which is unaffected by thelevel of current productivity. We use 3 different values of the TFP shock: negative shock, no shock and positive shock,corresponding with the factor of:

$$\hat{A} = \begin{pmatrix} 0.96696 \\ 1.00000 \\ 1.03304 \end{pmatrix} \tag{12}$$

In a broader sense, there is the possibility that A' might take on any number of discrete values, each of which would have a distinct probability. In this instance, A' complies with the specifications of a normal distribution. Because of the manner that we have defined the stochastic element, the distribution of A' is not reliant on the distribution of A. In other words, the effects of this completely external shock, denoted by the letter A, are only transient(Carlson *et al.*, 2012).

4.2. Choice of Grid Points and Initializing Algorithm

The dynamics of the Ramsey model require that the optimal sequence of capital stocks monotonically approaches the stationary solution K''determined from the condition $\beta f'(K^*)$ =1. When the shock to A is unknown as in the stochastic case, we make an educated guess to find a stationary value that would be reached if there is no shock to the system if A = 1. This we find K^* from the equation $1 = \beta(1-\delta + f'(K^*))$ which yields $K^* = (\alpha / (1/\beta - (1-\delta)))^{i^{1-\alpha}}$. We place our grid points in the region [0.8K^*,

$1.2K^*]x[0.96696, 1.00000, 1.03304]$. We start with 4 grid points between $[0.8K^*, 1.2K^*]$ and use the results to interpolate to $4*2$ points on the second iteration, $4*2^2$ points on the third iteration and so forth until the seventh and final interpolation round resulting in $4*2^6 = 256$ grid points.

At the beginning, we gave the value function a value of 0 to represent it. To calculate the value function on a grid consisting of 4 points, we used the iteration technique of value function computation. According to Heer and Maussner (2011)'s recommendation, we made use of this coarse grid in order to speed up the algorithm. Through a series of iterative processes, we brought the grid's point count up to 256. At each stage, we started with the previously determined value function, computed further estimates of the value function using linear interpolation, and then utilized the result as an initial prediction for the value function. We continued the experiment until we obtained a tolerance for error of 0.00005, which needed a maximum of 1000 iterations each step, dropping by around 100 iterations with each step until we converged on the last round after approximately 200 iterations. The total amount of time that the experiment was conducted was 2 hours 46 minutes. We tried out a variety of model calibrations, some of which were effective in reducing the amount of time required for the run (for example, when we used $\alpha = 0.33$, $\eta = 0.9$ and $\delta = 0.1$, the amount of time required for the run was significantly decreased). However, in order to keep the results comparable, we decided to utilize the values that were found in the research.

5. The Result

We begin by observing the policy function in **Figures 2**. In **Figure 2(a)** we plot the policy function for each value of K_t we plot the corresponding K_{t+1}. Although it looks like a 45 degree line, it is not. We note from the policy function thatwhen $K_t = 11.009$ for $A = 1$, $K_{t+1} = 11.447$ whereas when $K_t = 55.047$, $K_{t+1} = 54.879$. Thus we go from accumulatingcapital stock to shedding it. Because of this, there is now an extremely strong possibility that the stationary solution will be included into the grid. In addition, we may see consumption at **Figure 2(b)**.

We are going to calculate the impulse responses of our model so that we can compare them to the responses that are predicted based on empirical data.We will simulate a one period shock to the total factor productivity A and track how consumption C, capital K', and factor prices $f'(K) = A\alpha K^{\alpha-1}$ co-move with contractionary and expansionary shocks. Impulse responses are deviationsof the model's variables from their stationary solution. The policy function calculated in our model gave us the saddle pathto the

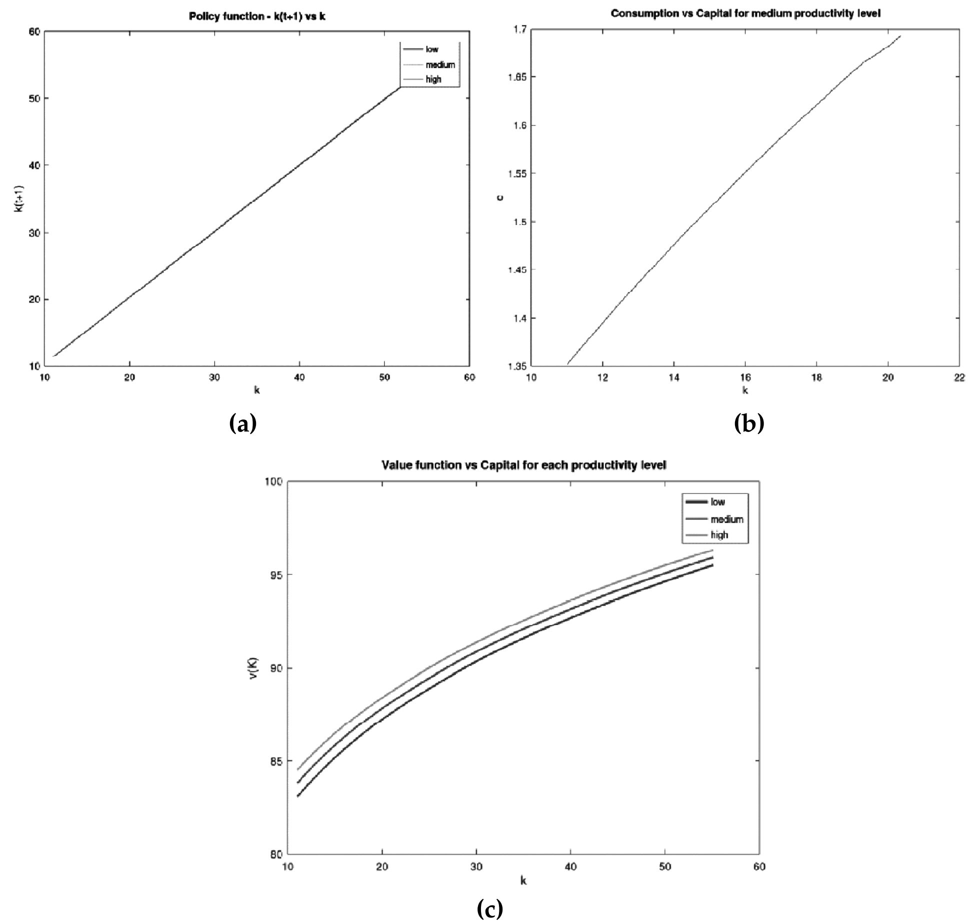


Figure 2: The Policy Function. In Figure 2(a), K′ is plotted against K and a constantly growing function is shown. We then exhibit C vs K in Figure 2 (b). In Figure 2(c), we examine the lifetime value functions for each level of beginning K, displayed for the three potential stochastic values of the TPF A. As anticipated, greater values of A result in greater lifetime values

stationary solution for each level of A. We need to verify what happens to a system when it gets an external shock that brings A to its higher level (1.03304) or its lower level (0.96696), comparing it to the stationary solution for $A = 1$.

The evaluation of the correctness of our policy function may benefit greatly from the usage of impulsive replies. Impulse reactions to a rapid and brief rise in total factor production may be shown in **Figure 3** thanks to a vector auto-regressive (VAR) model that was calculated from actual data by Heer and Maussner (2011). We are seeing knee-jerk reactions in terms of production, consumption, investment, and hours worked. Because log$A$ has a significant degree of autocorrelation (they also use ρ = 0.90), A_t is consistently greater than $A = 1$ during the numerous periods. People's real wages go up as a result of increased total factor productivity, which causes them to spend more money and put in longer hours at work at the expense of their leisure time. During the time period in which the shock is detected, greater production *y(K)* and increased labor supply both contribute to rises in factor prices. Together, these two factors raise productivity and increase the supply of workers. The family wishes to maintain a consistent level of consumption over time and is planning on higher interest rates (factor prices) in the future because they believe that the marginal product of capital has improved in a manner that is both long-term and stable. This, in turn, results in an increase in investment throughout the subsequent era. Because of the incremental but steady growth of future capital, real wages will continue to be relatively high even after working hours have been reduced to their traditional levels.

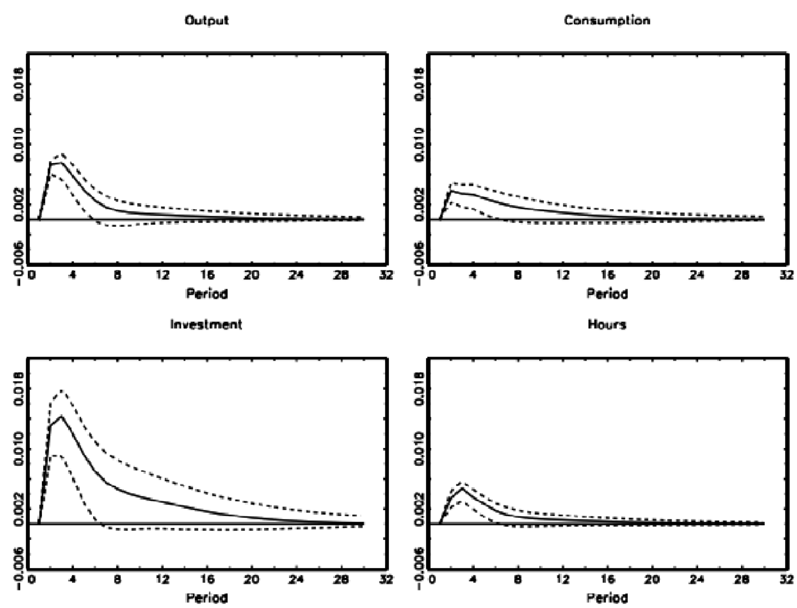


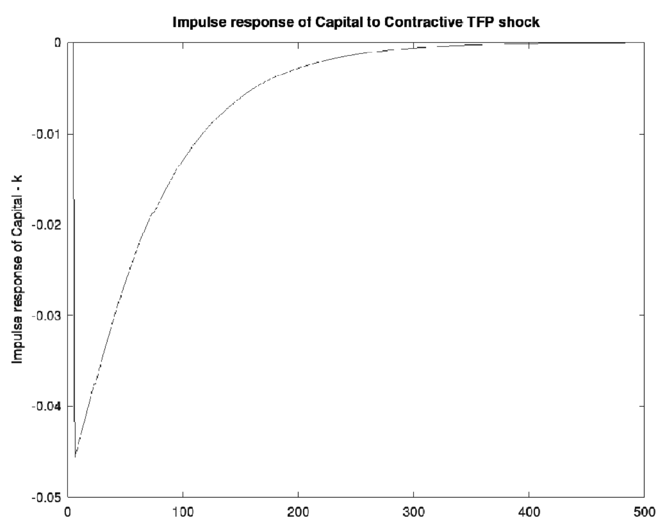
Figure 3: Impulse Responses from Estimated VAR

Since we have presumed that A is an entirely exogenous process, we do not utilize a VAR process to describe it when we plot our impulse responses. This is due to the fact that A is driven by a set of probabilities that are also exogenous. At time $t = 5$, we cause a shock to the value of A, and at time $t =$

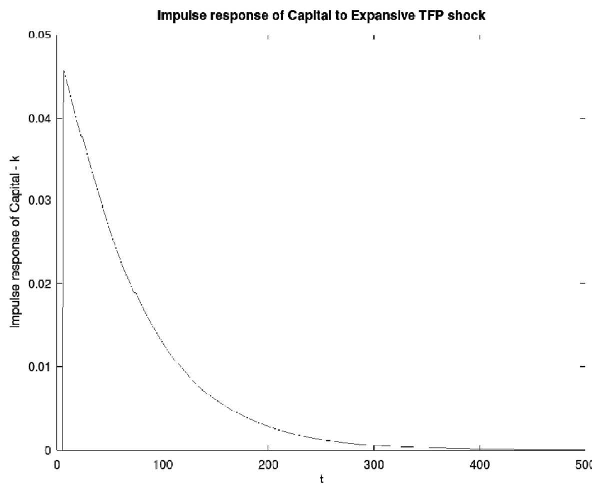
6, we reset the value of *A* to 1 (the initial value). Since this is not the same as the VAR model described before, we may anticipate that the consequences of the shock will have a shorter-lasting impact in our model compared to the actual data. Despite this, we anticipate that the broad trend of either rise or reduction will be correctly identified. Furthermore, we anticipate that any impacts on the accumulation of capital will be more long-lasting than the effects on the other variables.

5.1. Impulse Response of Capital

In **Figure 4(a)**, at $t = 5$, we witness the impulsive response of capital accumulation to a contractionary shock to *A*, in which *A* goes from 1 to 0.96696. At $t = 6$, *A* returns to 1. We compare it to the values for capital stock given by our value function for $A = 1$. These values correspond to the y-axis level 0. Despite the fact that *A* returns to its original value in the very next period, it is evident that the contraction in *A* causes an immediate and long-lasting reduction in capital stock. If the shock in *A* had been more chronic, the contraction in capital would have been more protracted as well. It takes more than 400 time periods for our model to ultimately disappear, indicating that it is plausible.Similarly in **Figure 4(b)** we observe the effect of an expansive shock to *A* where it goes from 1 to 1.03304 at $t = 5$. Areturns to 1 at $t = 6$. In concurrence with the increase in output seen in the **Figure 3**, we find a persistent increase in capitalaccumulation as well.Consequently, we may infer that our model well captures the impulsive reactions of capital to a shock in total factor productivity.



(a)



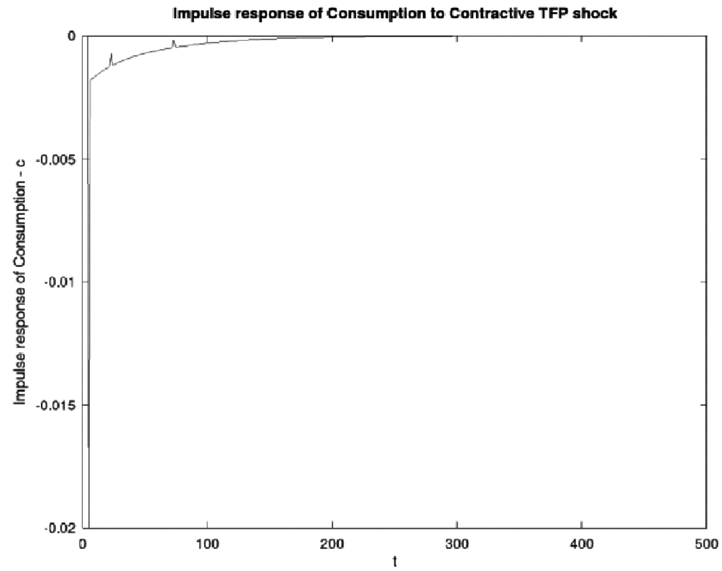
(b)

Figure 4. Impulse Response of Capital.We can see a contractionary shock to TFP shown in Figure (4a), which results in a decrease in production and, as a result, future capital accumulation. We see an expansive shock to TFP in Figure 4(b), which boosts output and persistently increases future capital accumulation permanently even after working hours return to normal. This is due to the fact that the increase in future capital is relatively small, but it remains even after the working hours return to normal

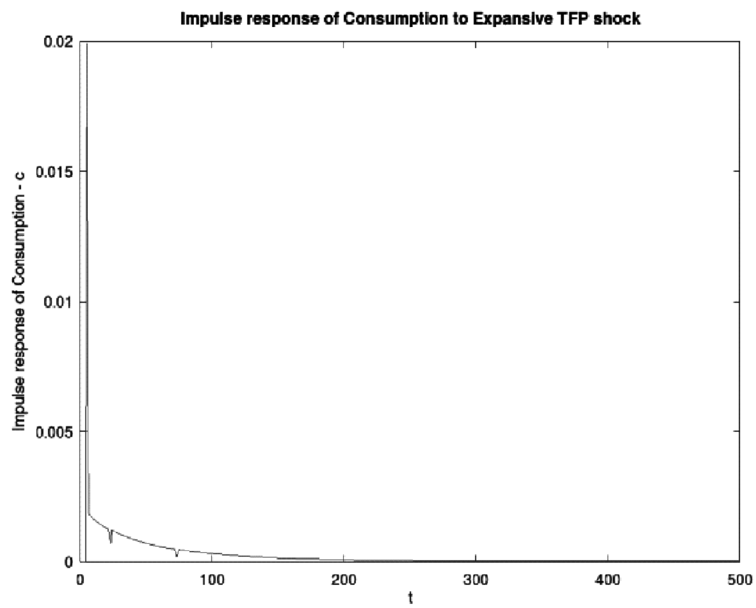
5.2. Impulse Response of Consumption

In **Figure 5(a)** we observe the impulse response of consumption to a contractionary shock to A where it goes from 1 to 0.96696at $t = 5$. A returns to 1 at $t = 6$. We compare it to the values of consumption generated by our value function for $A = 1$.The level 0 on the y-axis represents these values.We can observe that the contraction in A immediately leads to a contraction in consumption, which swiftly fades away when A returns to its original level in the subsequent period. If the shock in A had been more chronic, consumption would have contracted much more persistently. However, the reduction of consumption is less permanent than that of capital accumulation. This confirms the robustness of our concept since it is consistent with Ramsey model predictions.

Similarly in **Figure 5(b)** we observe the effect of an expansive shock to A where it goes from 1 to 1.03304 at $t = 5$.A returns to 1 at $t = 6$. In agreement with the increase in output seen in the **Figure 3**, we find a temporary increase inconsumption as well. As a result, we are able to reach the conclusion that our model has been effective in capturing the impulsive reactions of household consumption in response to a shock in total factor production.

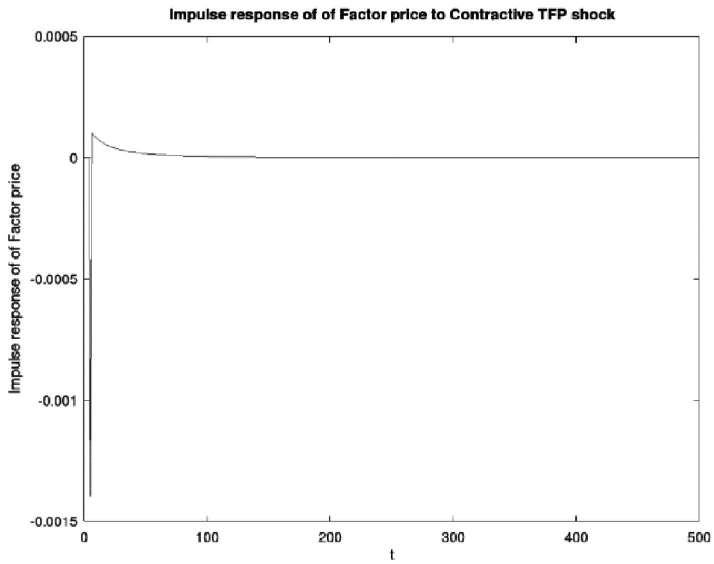


(a)

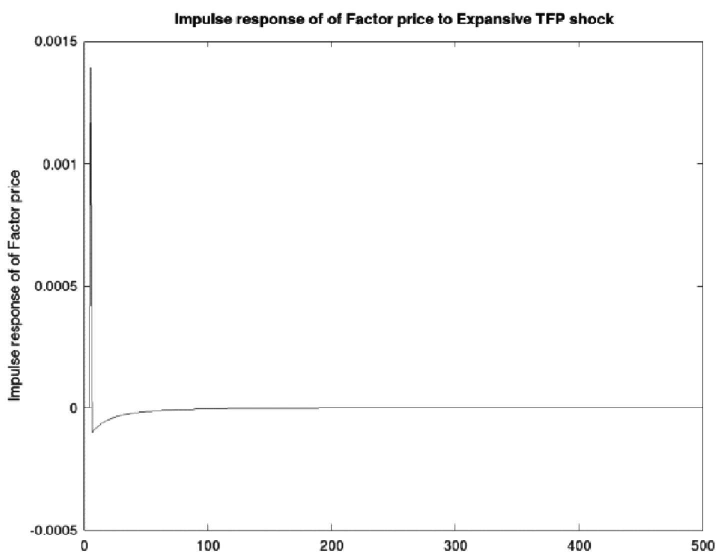


(b)

Figure 5: Impulse Response of Consumption.In Figure 5(a), we observe a contractionary shock to TFP that reduces output and therefore leads to a temporary dip in household consumption. In Figure 5(b), we observe an expansive shock to TFP shock that boosts output and consumption, although temporarily



(a)



(b)

Figure 6. Impulse Response of Factor Prices. In Figure 6(a), the effects of two shocks are depicted: a contractionary shock to A, which depresses factor prices, and an unanticipated return to normality, which causes interest rates to increase to make up for the preceding period's under-investment. In Figure 6(b), the opposite effect is shown, in which the marginal product of capital increases with productivity but falls momentarily when the system rapidly returns to normal

5.3. Impulse Response of Factor Prices

The factor prices in our model are unconstrained by any smoothing parameters and long-term considerations and thereforeaccurately reflect the unexpectedness of the series of shocks - one shock at $t = 5$, and the other sudden movement back to *status quo* at $t = 6$.

Figure 6(a) demonstrates that when TFP suddenly declines, the marginal product of capital falls and investment and consumption drop. In the very following period, however, when TFP returns to normal due of a period of under-investment, the marginal product of capital is now greater than its stationary values had the economy never experienced the shock. However, the effect wears off fairly soon. **Figure 6(b)** depicts the inverse of the preceding motion. When total factor productivity (TFP) rises, the marginal product of capital rises, leading to over-investment. This tendency is reversed in the very following period, when TFP returns to normal and the marginal product of capital is lower than it would have been if the aforementioned shock had not occurred. However, the impact is temporary in nature.

6. Conclusion

Because none of the values ofderived by the value function iteration surpass the upper limit of our grid point, a quick analysis of the policy function would indicate that it is not confined by the discrete state space.We areable to observe an increasing trend forgiven a small, as well as a decreasing trend for large values of initial capitalstock. The value function and policy function of our stochastic Ramsey model may be effectively captured by procedures that take place in discrete state spaces. In general, our model is able to provide results that are consistent with the actual data in broad strokes. Both the theoretical model and the data show that shocks to A cause capital, consumption, and factor prices to move together in the way that was expected by the model. When we simulate our reactions, we do not represent TFP as a VAR process, which results in a reduction in the shocks' persistence. Despite this, the model is able to accurately predict both the overarching trend of the shocks as well as their relative persistence across the variables that were investigated. There are many different ways that things may be improved.If we make a solid initial estimate for the value function, we will be able to significantly reduce the amount of time it takes for our algorithm to complete its tasks, such as the one used by Heer and Maussner (2011): . In addition, we may increase the accuracy of our results by using a less coarse starting grid in conjunction with cubic interpolation. Despite this, the performance of our current model is satisfactory considering the limitations imposed by time and computing resources.

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